Stop Being a Hound with the Union Bound
and Start Your Reunion with the Disjoint Union.

Suppose A, B and C are events.

1. Apply the union bound to $\Pr[A \text{ or } B \text{ or } C]$.
2. Show that $\Pr[A \text{ or } B \text{ or } C] = \Pr[A] + \Pr[B \text{ and } \neg A] + \Pr[C \text{ and } \neg B \text{ and } \neg A]$.
3. Suppose I flip 3 coins. What is the probability I flip at least one heads? Use the union bound (1) and then the sharper union bound (2).

Fast Sorting. Let $x_1, \ldots, x_n$ be numbers generated independent and uniformly at random from the range $[0, 1]$. We will give a randomized algorithm that sorts these numbers in expected time $O(n)$. (Hint for the algorithm: for any $i \in [n]$, what is the probability that x_i falls in the range $[0, .1]$? How about $[2/n, 3/n]$? Then, what is the expected number of $x_i's$ which fall in the range $[2/n, 3/n]$?)