

Problem (Excercise 17.2). Let $G = (V, E)$ be a directed graph. For $s, t \in V$, let $\lambda(s, t)$ denote the size of the minimum (s, t) -cut. Prove the following “triangle inequality” for cuts:

$$\lambda(a, b) \geq \min\{\lambda(a, c), \lambda(c, b)\}$$

for any three vertices $a, b, c \in V$.

Problem (Slop-Maxxing). Suppose there is a $n \times n$ grid with the letters S, L, O, and P. Your goal is to form as many *disjoint* copies of the word SLOP as possible from the grid with the following rule: each letter of SLOP must be consecutive, i.e., you can start at any S, move to a neighboring L, move to its neighboring O, and so forth¹. For example,

S	L	O	P	S
L	S	L	O	P
O	S	O	L	L
S	P	O	S	P
P	S	L	L	O

has solution

<u>S</u>	<u>L</u>	<u>O</u>	<u>P</u>	S
L	<i>S</i>	<i>L</i>	<i>O</i>	<i>P</i>
O	S	O	L	L
S	<u>P</u>	<u>O</u>	<u>S</u>	<u>P</u>
P	<u>S</u>	<u>L</u>	<u>L</u>	<u>O</u>

Give a polynomial time algorithm to achieve your goal of finding the most SLOP as possible.

¹Here, a letter Y is neighboring X if you can move up, down, left, or right once from X to get to Y , and vice versa.