PS0 2

The one where we make you work together.

Exercise I.2. For the following problems find the smallest values of a, b such that $S(n) = O(n^a \log^b(n))$.

$$S(n) = \sum_{i=1}^{n} (4/5)^i$$

$$S(n) = \sum_{i=1}^{n} \frac{3}{i}$$

$$S(n) = \sum_{i=1}^{n^2} \frac{7}{i^2}$$

$$S(n) = \sum_{i=1}^{\log_2(n)} (5/4)^i$$

$$S(n) = \sum_{i=1}^{n} \sum_{j=1}^{i^2} j$$

Cheat Sheet Sums

1) Geo. Decreasing $(0 < r < 1)$

$$\sum^{\infty} r = \frac{1}{1-r}$$

[redacted] Sum (diverging)

$= O(\log n)$

(converging)

$= \pi^2/6 = O(1)$

$(r \geq 1)$

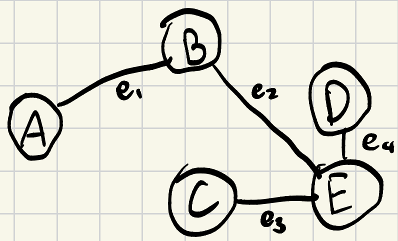
$$\frac{1 - r^m}{1-r}$$

Sum

$$\frac{n(n+1)}{2}$$

2. `min-vertex-cover`$(G = (V, E))$: Given an undirected graph $G = (V, E)$, return the minimum size of any vertex cover of G.

(A vertex cover is a set of vertices $S \subseteq V$ that includes at least one endpoint of every edge $e \in E$.)



Guiding Questions

1) What is the min cover for this example?

2) Suppose I have 2 vertex covers C_1 and C

   One of them is guaranteed to be the minimum. How do I pick the minimum vertex cover given C_1 and C_2?

3) (Base Case) For what graphs G can I immediately return the min cover?

4) (Induction Step) Based on (2), if I can find such C_1 and C_2 that come from 'smaller' graphs then I can do induction.

Exercise I.3. Prove by induction that a tree with n vertices has $n-1$ edges.

We prove proposition $p(n)$ = any tree T with n vertices has $(n-1)$ edges.

BC

IH:

IS: Let T be a tree with _ vertices.

Exercise 1.7. Let $A[1..n] \in \mathbb{R}^n$ be an array of n numbers. An *inversion* is a pair of numbers out of increasing order; more precisely, a pair of indices $i, j \in [n]$ such that $i < j$ and $A[i] > A[j]$. Design and analyze an algorithm (as fast as possible) for counting the number of inversions in A.

$[3\ 2\ 5\ 4\ 1\ 7]$

1) # inversions in ex?

2) General Technique for Solving?

3) How do I count inversions given two sub problems?